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On Some Difficulties in the Theory of Transfinite Numbers and Order Types. B. RUSSELL. *Proceedings of the London Mathematical Society,* Series 2, Vol. IV., Part 1. Pp. 29–53.

This paper, which was suggested by a paper of Dr. Hobson,[1] aims at a generalization of such contradictions as those of Burali Forti[2] and of Russell[3] and at the introduction of certain distinctions through which Mr. Russell hopes to be able to overcome the present difficulties in the foundations of mathematics.

As to the contradictions, three theories are proposed as offering possible modes of escape. They are (*a*) the zigzag theory, (*b*) the theory of limitation of size, (*c*) the no-classes theory. With each Mr. Russell also presents the difficulties peculiar to it. Thus, the zigzag theory maintains that certain complicated functions do not define classes at all, but we do not know which do and which do not, and the axioms determining this question are very complex. The theory of limitation of size asserts that excessive size is what a class must avoid. Where shall we stop? Is ω illegitimate and are all proper classes finite? And, lastly, the no-classes theory takes the extremely radical standpoint that there are no such existences as classes, relations and functions. Strangely enough, although this view is opposed, as Mr. Russell says, by common sense, although it destroys a great part of Cantor's work, and its working out is so extremely complicated that we are not sure it leaves us even enough for arithmetic, yet at the end of the paper we find a curious paragraph which reads: "From further investigation I now feel hardly any doubt that the no-classes theory affords the complete solution of all the difficulties stated in the first section of this paper."

It may be that this note will be justified, but as we read over Mr. Russell's statement of the contradictions (§ 1) there seem to be some neglected aspects of the problem which also point to a solution, simpler and more complete than any he has suggested. For example, the words *norm, property* and *propositional function* are used synonymously (p. 30), but we shall see that norms do not always stand for properties. Thus the contradiction as to cardinals consists in asserting that there is some property such as '$x = x$' common to all entities, and the cardinal number so defined must be the greatest of all cardinal numbers. But Cantor has proved that there is no greatest cardinal number, hence a contradiction. If we accept Aristotle,[4] however, we can not admit that there is any property common to objects denoted by negative norms such as 'X is a not-man.' Lotze[5] is emphatic upon this point. There is, then, no property common to all things. The selection here is certainly unfortunate, for if we interpret '$=$' with Dedekind,[6] '$x = x$' merely means that we have written twice the name of the thing, and one does not have to be even as much of a realist as Mr. Russell (p. 41) professes to be to deny that the number of times the name of an object may be written is any property of the object. The fact that all the contradictions make use of these negative norms is significant. Such a norm is legitimate *as a denoting phrase for an aggregate of terms,* but because of the fact that these terms have no property in common, we can not treat them as though there was a corresponding class concept; the class as one does not exist. So long as the aggregate is dealt with as a mere denoted extension the contradictions do not arise. Hence, although there is no difficulty in using norms and propositional functions as synonymous, some norms do not stand for properties.

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[1] Hobson, *Proceedings of the London Mathematical Society,* Series 2, Vol. III., pp. 170–188.

[2] Burali Forti, 'Rendiconti der Circulo Mathematico di Palermo,' 1897.

[3] Russell, 'Principles of Mathematics,' Ch. X., also Appendix B, § 500.

[4] Aristotle, 'Hermeneutica,' Ch. II.

[5] Lotze, 'Logic' (Eng. trans.), p. 48.

[6] Dedekind, 'Was sind und was sollen die Zahlen,' p. 1.

Another assumption contained in the development of the contradictions is that a class may be contained in itself as an element. Schönflies[7] has already objected to this, without, however, making the grounds for his objection very forceful. When a class is an aggregate denoted by a negative norm the objection is evidently justified, for the class, in the only sense in which it could be an element, does not exist. The objection is equally justified when the class is defined by a property. In pure mathematics it is the logical and not the empirical extension of a class which is in question; its extension may be obtained from the class concept by logical division. A class, to be an element of itself, must then be at once identical with and different from itself;[8] or from a slightly different point of view we may say that the judgment '$x \epsilon x$' is particular, and, as Schröder[9] has already pointed out, in the logic of intentions particulars are *nichtssagende.* In previous work also Mr. Russell has confused logical and empirical extension, and neglected to exclude the latter from pure logic.[10]

The complete implications of these distinctions are worth developing. They free us at least from the generalized type of contradiction given by Mr. Russell (p. 35) and put us in harmony with Cantor's result that there is no greatest cardinal number. They also agree with Mr. Russell's statement (p. 36) that there are some classes such that we can never pretend to collect all their terms, for there will always be a term not in any collection we may have made, although the class is merely simply infinite. This fact seems to me more significant for the theory of the transfinite than Mr. Russell appears to observe.

The last part of Mr. Russell's paper is devoted to a discussion of the axiom on which depends Zermelo's proof[11] that every class can be well ordered, and the related multiplicative axiom. The problem is to prove the existence of a *norm* by which we can pick out one term from each class in any class of classes. On the validity of these axioms depends the possibility of proving the identity of inductive and finite numbers and of establishing the laws of addition, multiplication and exponentation. They may be true only under certain conditions not yet defined; and with respect to transfinite classes, they are especially under a cloud, the lightening of which, we believe, will come from some such logical considerations as have here been suggested.

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[7] Schönflies, *Jahresbericht der Deutchen Mathematiker-Vereinigung,* Bd. 15, January, 1906.

[8] Cf. Schönflies, *loc. cit.*

[9] Schröder, 'Algebra der Logik,' Bd. 1, p. 100.

[10] Cf. Remarks as to extension of class 'men' and 'featherless biped,' 'Principles of Mathematics,' § 24.

[11] *Mathematische Annalen,* Bd. 59, pp. 514–516.

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JOURNALS AND NEW BOOKS

REVUE PHILOSOPHIQUE. March, 1906. *Le Mensonge du Monde* (pp. 233–267): PAULHAN. – A resumé of writer's metaphysics. The whole universe is a liar incorrigible because all harmony manifested in physical, biological and mental organizations is fictitious, serving only to conceal the real conflict of the elements involved. Harmony is only relative, for all systems, atomic and social alike, are built up to serve certain purposes common to all the elements in the systems; this community of interest is of much less significance than individual differences. When the common end is attained, the system dissolves, *e. g.*, a committee or an organism. Finally, every effort toward an end involves the construction of a fictitious world which is harmonious only with the desire, not with the totality of facts; all religion, politics and personal ambition create such fiction unavoidably. *Sur la philosophie de Renouvier* (pp. 268–293): PILLON. – A critical review of Séailles's presentation of Renouvier's philosophy. Renouvier is shown to have passed through three distinct periods of belief: first, pantheism, then rational phenomenalism and, finally, critical phenomenalism. *Le caractère et le tempérament* (pp. 294–300): RIBÉRY. – In order to get a scientifically useful definition of temperament one must determine what the physiological phases of individuality are. Many students of temperament place too much emphasis upon circulatory and other vegetative processes. "The temperament depends upon the way the nervous system receives external and internal stimuli." 'The time is ripe for special studies of temperament' which shall supplant classifications. *Notes et documents: Hystérie et mysticisme* (pp. 301–308): MONTMORAND. – A study of the case of St. Theresa. *Revue critique:* Fouillée, *Le moralisme de Kant et l'amoralisme contemporain:* J. SEGOND. *Analyses et comptes rendus:* Picard, *La science moderne et son état actuel:* ABEL REY. Duhem, *La théorie physique, son objet et sa structure:* ABEL REY. Ball, *Histoire des mathématiques:* JULES SAGERET. Guastella, *Saggi sulla teoria della cognoscenza Saggio secondo: filosofia della metafisica:* J. SEGOND. Morselli, *Principi di logica:* G. L. DUPRAT. Dumas, *Psychologie de deux messies positivistes, Saint-Simon et Auguste Comte:* G. R. D'ALLONNES. Elkin, *Hume: The Relation of the Treatise*